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How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

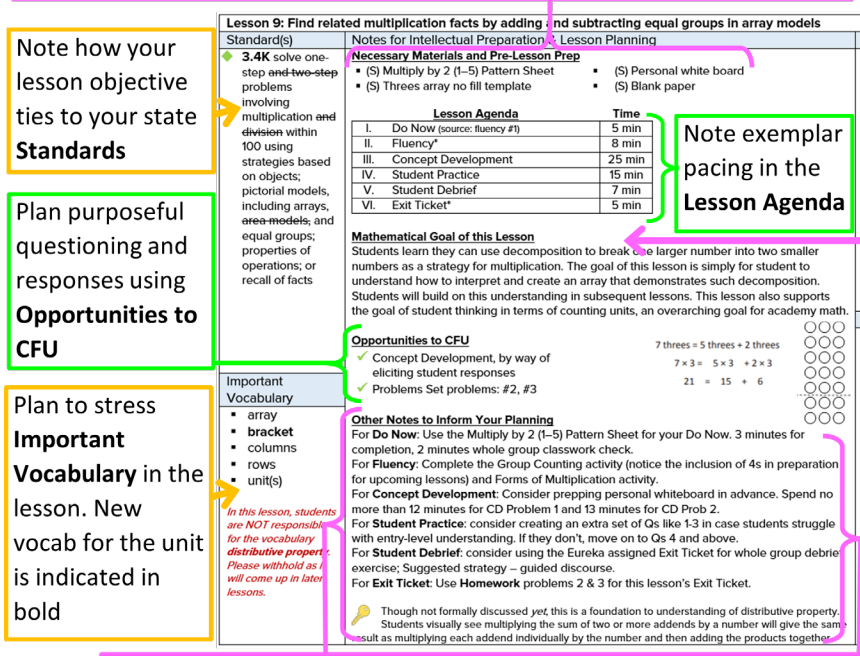
Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold



Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models Date: _____

Standard(s)
3.4K solve one-step ~~and two-step~~ problems involving multiplication ~~and division~~ within 100 using strategies based on objects; pictorial models, including arrays, ~~area models,~~ and equal groups; properties of operations; or recall of facts

Important Vocabulary
- array
- **bracket**
- columns
- rows
- unit(s)

*In this lesson, students are NOT responsible for the vocabulary **distributive property**. Please withhold as it will come up in later lessons.*

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep
- (S) Multiply by 2 (1–5) Pattern Sheet
- (S) Threes array no fill template
- (S) Personal white board
- (S) Blank paper

Lesson Agenda		Time
I.	Do Now (source: fluency #1)	5 min
II.	Fluency*	8 min
III.	Concept Development	25 min
IV.	Student Practice	15 min
V.	Student Debrief	7 min
VI.	Exit Ticket*	5 min

Mathematical Goal of this Lesson
Students learn they can use decomposition to break one larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

Opportunities to CFU
- ✓ Concept Development, by way of eliciting student responses
- ✓ Problems Set problems: #2, #3

7 threes = 5 threes + 2 threes
7 × 3 = 5 × 3 + 2 × 3
21 = 15 + 6

Other Notes to Inform Your Planning
For **Do Now**: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.
For **Fluency**: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.
For **Concept Development**: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.
For **Student Practice**: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.
For **Student Debrief**: consider using the Eureka assigned Exit Ticket for whole group debrief exercise; Suggested strategy – guided discourse.
For **Exit Ticket**: Use **Homework** problems 2 & 3 for this lesson's Exit Ticket.

Though not formally discussed *yet*, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

Lesson Look Fors
Look for teachers to...
- ❑ Have established a signaling routine for choral response or work show during the respective fluency activities
- ❑ Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array
- ❑ Make the focus of the lesson understanding the visual representations

Look for students to...
- ❑ Explain what they see in the array and how it relates to a given number sentence.

Student Criteria for Success
- Know: Shading, brackets, and/or dotted lines on an array will have mathematical significance
  -brackets can identify parts or wholes
  -dotted lines and shading represent decompositions
- Know: We count units; In an array, counting rows is the same as counting units.
- Know: Addition/subtraction and multiplication math facts (up to 4)
- Do: Interpret an array
  -identify decompositions within an array
  -Relate an annotated or labeled array to one or more number sentences
- Do: Addition/subtraction (+/- up to 4) Multiplication (2, 3, and 4)

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS

In Algebra 1, students were introduced to polynomial operations including polynomial long division. In this unit, students will further classify and define higher order polynomials and make judgments about key components of their graphs, including zeros, relative extrema, and end behavior. Students will perform operations on polynomials and will simplify a variety of expressions using adding, subtracting and division of polynomials. Polynomial division is used to determine exact factors and zeros of polynomial functions as well. Students will factor sum of cubes and difference of cubes. Students will utilize the Fundamental Theorem of Algebra to determine zeros of given polynomial functions. Through the Fundamental Theorem of Algebra, students will explore complex numbers and utilize complex conjugates to understand the number and types of zeros of a polynomial function.

Solving Equations & Inequalities: Solving an equation is the process of rewriting the equation to make what it says about its variable(s) as simple as possible.

- You can add, subtract, and multiply two polynomials. The result is always another polynomial, so the set of polynomials is closed under addition, subtraction, and multiplication. In this respect, polynomials are like the integers, which share the same closure properties.

Equivalence: A single quantity may be represented by many different expressions.
The facts about a quantity may be expressed in many different equations.

- You can divide polynomials using steps that are similar to the long division steps that you use to divide whole numbers.
- The factors of the numbers a_n and a_0 in $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_1 x + a_0$ can help you factor $P(x)$ and solve the equation $P(x) = 0$.
- The degree of a polynomial equation tells you how many roots the equation has.

Functions: A function is a relationship between variables in which each value of the input variable is associated with a unique value of the output variable. Functions can be represented in a variety of ways, such as graphs, tables, equations, or words. Each representation is particularly useful in certain situations. Some important families of functions are developed through transformations of the simplest form of the function.

- A polynomial function has distinguishing "behaviors." You can look at its algebraic form and know something about its graph. You can look at its graph and know something about its algebraic form.
- Finding the zeros of a polynomial function will help you factor the polynomial, graph the function, and solve the related polynomial equation.
- If $(x - a)$ is a factor of a polynomial, then the polynomial has value 0 when $x = a$. If a is a real number, then the graph of the polynomial has $(a, 0)$ as an x-intercept.

Misconceptions:

- When subtracting polynomials, some students may only distribute the negative to the leading coefficient instead of all of the terms of the polynomial being subtracted.
- Students may think that all zeros of a polynomial can be found by looking at the graph. There may also be complex zeros that are not x-intercepts.
- Students may think that complex conjugates always mean $a - bi$, where in reality it means the pair $a + bi$ *and* $a - bi$.
- Students may think that the Fundamental Theorem of Algebra guarantees real zeros when it only guarantees complex zeros. Even degree polynomials may have no real zeros.

Underdeveloped Concepts:

- Students may struggle with factoring which may make dividing polynomials more challenging.
- Many students do not know the traditional long division algorithm from elementary school, and it should not be assumed that they know how to divide long division or how to display a remainder as a fraction.
- Students may think that synthetic division and polynomial long division are interchangeable, where synthetic division only works with a linear divisor with a leading coefficient of 1.
- The terms Roots and Zeros can be used interchangeably, and both directly relate to the term Factors, but Factors is not an equivalent term.

Key Questions:

- How can the key attributes of polynomial functions be used to make predictions and critical judgments?
- How does the degree of a polynomial function affect the end behavior of the graph?
- How can the degree of a polynomial function be used to determine the potential number and types of zeros of the polynomial?
- How can the number and types of zeros of a polynomial function be determined? Algebraically and graphically.

CONTENT STANDARDS

Below are the standards addressed in this unit.

Readiness Standards	Supporting Standards
2A.2A Graph polynomial functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.	**2A.7B** Add, subtract, and multiply polynomials.
	2A.7C Determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two.
2A.7E Determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping.	**2A.7D** Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods.
NOTE: Students will also work with higher degree polynomials and their graphs in accordance with vertical alignment and AP readiness goals.	**2A.8A** Analyze data to select the appropriate model from among linear, quadratic, and ~~exponential~~ cubic models.

Focus on Disciplinary Literacy 	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → / Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓			✓							
makes use of graphic organizers	Graphic Organizers	✓			✓		✓	✓				
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives											
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓	✓		✓		✓		✓	✓	✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓		✓		✓		✓				✓
includes strategies that support language development												
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share											
includes sentence stems to support students with explanations	- Sentence Stems											
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics											
includes websites or equipment that enhances the lesson	Technological Support		✓	✓		✓					✓	✓
content can be presented in different forms												
uses hands-on tools or manipulatives to represent the math	- Concrete											
uses drawings to represent the math	- Pictorial	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

The EFFL Model

Before You EFFL!

Here are helpful resources that you guide you in the right direction before your EFFL lesson!

Why Should We EFFL?

The article advocates for the Experience First, Formalize Later (EFFL) teaching model, emphasizing its effectiveness in fostering deep understanding and flexible thinking in students. The author compares traditional teaching to a game of "Simon Says," where students merely mimic instructions without grasping underlying concepts. In contrast, EFFL encourages students to engage actively with problems, enhancing their ability to understand and apply math concepts creatively.

Tips for Lesson Planning

The article offers practical advice for effective lesson planning beyond the exhaustive and overly detailed approaches often emphasized during teacher training. It underscores the importance of thoughtful preparation but rejects the notion that teachers need to script every minute or detail of a class session.

Making the Most of Your EFFL Lesson Debrief

The article discusses the significance of the debriefing phase in the Experience First, Formalize Later (EFFL) lesson model, emphasizing its role in reinforcing learning and highlighting student contributions. The debrief session is seen as crucial for integrating academic vocabulary, emphasizing key lesson understandings, and valuing students' mathematical insights.

While You EFFL!

While each lesson may be unique in context and skills, all lessons benefit from the following practices:

Teacher Look Fors:

- ❑ Utilizing the Do Now to spark students' interest in the Activity.
- ❑ Use questioning to promote small group discussion and exploration, guided by monitoring questions.
- ❑ Connects Experience First to formal concepts using a colored pencil/pen to take notes along the margin during the Debrief.
- ❑ Facilitates whole-class discussions for students to reflect, share insights, and provides feedback that reinforces key concepts.
- ❑ Tracks time to adapt lesson pacing and support based on student response and engagement.

Students Look Fors:

- ❑ In the Activity, students engage in group work and discourse.
- ❑ Exploring the activity, testing hypotheses and approaches (trial & error).
- ❑ Take notes on key ideas and concepts using different colored pencil/pen to take notes along the margin.
- ❑ Share thoughts and ideas that demonstrate their approach to their work.

Other considerations

- During the **Experience First** phase, if most of your students seem stuck or disengaged, take a moment to pause, reset, and provide clear instructions. Some problems of the Activity are more suitable to do a whole-class discussion as a means to save some instructional time for Student Practice or the Exit Ticket. You are encouraged to adapt the EFFL (Experience First, Formalize Later) process to meet your students' needs while maintaining a focus on student-centered instruction.

ROADMAP

AT A GLANCE: Unit 7 – Polynomials			
Day	Date	Lesson	Lesson Title
1		1	What is a Polynomial?
2		2	Forms of Polynomial Equations
3		3	Polynomial Function Behavior
4		4	Multiplying and Dividing Polynomials
5		5	Factoring Polynomials
6			*Unit 7 Success Day 1 – Review & Reteach topics from 7.1 – 7.5*
7		6	Polynomial Long Division
8		7	Polynomial Synthetic Division and Factor Theorem
9		8	Using Sum/Difference of Two Cubes & Grouping to Find Polynomial Factors
10		9	Fundamental Theorem of Algebra
11		10	Graphing Polynomial Functions
12			*Unit 7 Success Day 2 – Review & Reteach topics from 7.6 – 7.10*
13		11	Solving Polynomials
14			*Unit 7 Success Day 3 – Use as needed based on your data*
15			*Unit 7 Success Day 4 – Unit Assessment Review*
16			End of Unit 7 Assessment

Date: _____

Lesson 1: What is a Polynomial

Standard(s)

2A.8A Analyze data to select the appropriate model from among linear, quadratic, and ~~exponential~~ cubic models.

Important Vocabulary

- **polynomial**
- variable
- term
- **monomial**
- **binomial**
- **trinomial**
- degree
- coefficient
- constant
- leading coefficient

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (10 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will determine if an expression is a polynomial and if so, identify the degree. Students will use common differences of output values to find the degree of a polynomial based on patterns they investigate.

Opportunities to CFU

- ✓ INM #2-3
- ✓ Debrief
- ✓ Student Practice #1-3

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

INM & Student Practice

A major goal of the unit is to connect what we have learned about linear and quadratic functions and apply it to higher degree polynomials. In this lesson we will look at some visual patterns, create sequences, and look for a common difference. What we want students to notice is that the number of times they have to take the difference until they get a common difference tells them the degree of the polynomial. The first pattern is a linear relationship so they will find a common difference after taking the difference between terms one time. The second visual pattern or sequence describes a quadratic function so it will take finding the difference between the terms and then finding the difference of the differences to get to a common difference. The directions explaining how to take the differences can be a little confusing so double check that students are doing this correctly as you are checking in with the groups.

Lesson Look Fors

Look for teachers to…

- ❑ Have students consider the differences between terms and the differences between the differences.
- ❑ Help students connect where the visual is in the pattern as the input (1^{st}, 2^{nd}, 3^{rd},…) and the number of blocks in each visual as the output.

Look for students to…

- ❑ Identify the change in the number of blocks as the difference and find the differences between the differences until they find a common difference.
- ❑ Associate the order of the differences needed to find a common difference with the degree of a polynomial.

Student Know/Do Chart

Students can

Do: Identify if an expression is a polynomial and determine the degree if it is.

Know: Polynomials can be classified by the degree (linear, quadratic, cubic, quartic, quintic,…) or the number of terms (monomial, binomial, trinomial, polynomial).

Date: _____

Lesson 2: Forms of Polynomial Equations

Standard(s)

- **2A.2A** Graph polynomial functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.
- **2A.7E** Determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping.

Important Vocabulary

- polynomial; general form of a; factored form of a

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (15 min)
- Debrief (10 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will find the x- and y- intercepts of a polynomial written in general or factored form. Students will also use the x-intercepts of a polynomial to write an equation for the polynomial.

Opportunities to CFU

- ✓ INM #1-2
- ✓ Debrief
- ✓ Student Practice #1-4

Other Notes to Inform Your Planning

The goal of today's lesson is for students to take what they learned about the general and intercept forms of a quadratic equation and to apply it to polynomials. Students should be able to work through the entire activity in their groups before debriefing as a class. They'll begin with a quadratic function. Students will take the intercept form of the quadratic and turn it into general form, graph the function, and identify how the intercepts of the function can be seen in the different forms. This is all review from Unit 4. We will build from this review to develop graphs and forms of polynomials in general.

Focus on Disciplinary Literacy

INM

Lesson Look Fors

Look for teachers to...

- ❑ Have students use Desmos to aide in graphing the given functions. Focus them on the intercepts.
- ❑ Help students connect the factored form of polynomials with the zeros.

Look for students to...

- ❑ Graph a polynomial and identify the x- and y-intercepts.
- ❑ Write a polynomial function in both general and factored forms.

Student Know/Do Chart

Students can

Do: Find the x- and y- intercepts of a polynomial written in general or factored form.

Do: Write the equation of a polynomial from the x-intercepts of its graph.

Know: A polynomial equation can be written in the form $y = ax^n + bx^{n-1} + \cdots + c$, where their y-intercept is $(0, c)$.

Date: _____

Lesson 3: Polynomial Function Behavior

Standard(s)

- **2A.2A** Graph polynomial functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.
- **2A.7D** Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods.

Important Vocabulary

- polynomial; general form of a; factored form of a
- degree
- leading coefficient
- end behavior
- **repeated zero**
- zero(s)

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (10 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be given a polynomial, determine the maximum number of x-intercepts, turns, and the end behavior based on the leading coefficient and the degree. Students will also use the degree of a factor to determine if the graph will touch or cross the x-axis at the associated x-intercept.

Opportunities to CFU

- ✓ INM #2-6
- ✓ Debrief
- ✓ Student Practice #1-6

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

INM & Student Practice

We want students to look at equations of polynomial functions and their graphs and to look for patterns. To do this most efficiently, we're going to give students one even degree and one odd degree polynomials to investigate. We want students to learn about how the degree of a polynomial determines the maximum number of x-intercepts for the function and how that shapes the end behavior. Later in the lesson we will investigate why some polynomials have fewer x-intercepts than the degree and some have the same.

Lesson Look Fors

Look for teachers to…

- ❑ Utilize graphing calculators or Desmos to support students in analyzing polynomial function graphs.
- ❑ Focus students on what the degree of the polynomial tells them about the shape of the graph.

Look for students to…

- ❑ Graph a variety of polynomial functions.
- ❑ Analyze and describe the key features of a polynomial graph and connect the equation to the intercepts and end behavior.

Student Know/Do Chart

Students can

Do — Determine the maximum number of x-intercepts, turns, and the end behavior based on the leading coefficient and the degree.

Do — Use the degree of a factor to determine if the graph will touch or cross the x-axis at the associated x-intercept.

Know — Polynomial function's end behavior is determined by the degree and leading coefficient. End behavior falls into one of four types: ↙ & ↗; ↖ & ↘; ↖ & ↗; ↙ & ↘.

Date: _____

Lesson 4: Multiplying and Dividing Polynomials

Standard(s)

- **2A.7B** Add, subtract, and multiply polynomials.
- **2A.7C** Determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two.

Important Vocabulary

- distributive property
- dividend
- divisor
- factor(s)
- product
- quotient
- remainder
- tabular method

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (15 min)
- Debrief (10 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will multiply polynomials of any degree and simplify the product. Students will also divide polynomials using a rectangle diagram and determine if they divide evenly or not.

Opportunities to CFU

- ✓ INM #1-3
- ✓ Debrief
- ✓ Student Practice #1-5

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

Do Now, INM, & Student Practice

In Unit 4 while studying Quadratics, students used rectangle diagrams or area models many times to multiply binomials, factor expressions and to complete the square. Today we're going to use them to multiply and divide polynomials. Multiplying the polynomials should be familiar to students. The only change from Unit 4 is that they will be multiplying polynomials with higher degrees. The new idea today is using a rectangle model to divide polynomials.

We want students thinking about multiplication and division as they did in elementary school. We want the students to think about how the parts of a division problem can be rearranged to create a multiplication problem.

Lesson Look Fors

Look for teachers to…

- ❑ Use the guiding questions to support students to build on what they did with quadratics in unit 4.
- ❑ Connect for students how the tabular method can be used to multiply or divide polynomials no matter the degree of each term.

Look for students to…

- ❑ Use the tabular method to divide polynomials as rectangles using the divisor as the first factor.

Student Know/Do Chart

Students can

Do — Multiply polynomials of any degree and simplify the product.

Do — Divide polynomials using a rectangle diagram and determine if they divide evenly or not.

Know — When dividing polynomials, like dividing integers, if the original polynomial can be written as multiplication of polynomial factors, then it divides evenly. If not, there would be a remainder.

Date: _____

Lesson 5: Factoring Polynomials

Standard(s)

- **2A.7E** Determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping.
- **2A.7D** Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods.

Important Vocabulary

- factor(s) (noun and verb)
- factoring
- product
- quotient
- remainder
- tabular method
- reverse tabular method

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (10 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will determine if an expression is a factor of a polynomial through division and if it is, find all remaining factors. Students will then factor a polynomial completely.

Opportunities to CFU

- ✓ INM problems #3-4
- ✓ Debrief
- ✓ Student Practice #1-5

Other Notes to Inform Your Planning

In previous lessons, students have learned about how the factored form of a polynomial shows the x-intercepts of the polynomial. In the last lesson, they were reminded that a factor of a number means that the factor divides the number evenly. For example, 6 is a factor of 24 because 24 ÷ 6 = 4. Additionally, we know from this division that 4 is also a factor. We're going to use these concepts today to guide students to an understanding that if a polynomial divides another evenly then it is a factor. We can use polynomial division to find the factors of a polynomial which helps us to factor a polynomial or to write it in factored form. Also, as we are finding the factors, we are finding the x-intercepts of the polynomial.

Focus on Disciplinary Literacy

INM & Debrief

Lesson Look Fors

Look for teachers to…

- ❑ Connect factoring integers to factoring polynomials and build from what students know how to do from unit 4.
- ❑ Help students make the connection between factors and x-intercepts so they can use the graph of a polynomial to look for potential factors.

Look for students to…

- ❑ Use the graph of polynomials to find factors and the tabular method to breakdown the whole polynomial into factors.
- ❑ Use division to decide if a given polynomial is a factor of a larger polynomial by deciding if it divides evenly.

Student Know/Do Chart

Students can

Do — Determine if an expression is a factor of a polynomial through division and if it is, find all remaining factors.

Do — Factor a polynomial completely.

Know — The tabular method can be used to divide two polynomials and find lower degree polynomials that may also need factored further.

Date: _____

Lesson 6: Polynomial Long Division

Standard(s)

- **2A.7C** Determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two.

Important Vocabulary

- dividend
- divisor
- quotient
- remainder
- divides evenly
- **placeholder**
- long division algorithm

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will divide polynomials using long division.

Opportunities to CFU

- ✓ INM problem #1-4
- ✓ Debrief
- ✓ Student Practice #1-4

Other Notes to Inform Your Planning

In this lesson, students will use their understanding of long division (divide, multiply, subtract, bring down) of numbers to divide polynomials. Some students may have not seen traditional long division algorithm in elementary school; you may need to first review with real numbers. Students will use their understanding of polynomials and degrees of terms to write the quotients of polynomials after dividing.

Lesson Look Fors

Look for teachers to…

- ❑ Connect long division of real numbers to polynomial long division and establish the algorithm for students.

Look for students to…

- ❑ Divide polynomials using the long division traditional algorithm.
- ❑ Represent the remainder as both the remainder term or as the rational expression using the divisor as the denominator.

Student Know/Do Chart

Students can

Do Divide polynomials using long division and state the remainder if there is one.

Know Know the long division algorithm and how to represent the remainder of polynomial division.

Date: _____

Lesson 7: Polynomial Synthetic Division and Factor Theorem

Standard(s)

- **2A.7C** Determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two.
- **2A.7D** Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods.

Important Vocabulary

- coefficient
- constant
- dividend
- divisor
- quotient
- remainder
- **synthetic division**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will divide polynomials by linear factors using synthetic division and apply the Factor Theorem to check for polynomial factors.

Opportunities to CFU

✓ INM parts B,C,D
✓ Debrief
✓ Student Practice #1-4

Other Notes to Inform Your Planning

Students will apply their understanding of long division with polynomials to divide using synthetic division. Students will use $x = a$, when $x - a$ is the divisor to divide. Students will use the remainder theorem to decide if $x - a$ is a factor of the polynomial. Students will continue to use their understanding of dividing to do various types of factoring with polynomials.

Lesson Look Fors

Look for teachers to…

❑ Support students as they set up synthetic division as it may be new and it is easy to get confused with the signs.

Look for students to…

❑ Determine if a polynomial division problem can be divided using synthetic division.
❑ Accurately set up and use synthetic division for a variety of problems.

Student Know/Do Chart

Students can

Do: Use synthetic division to divide polynomials by linear factors of the form $x - a$.

Do: Use the Factor Theorem and Remainder Theorem with synthetic division to determine if $x - a$ is a factor of the polynomial.

Know: If the remainder of synthetic division by $x - a$ is 0, then $x - a$ is a factor of the polynomial.

Date: _____

Lesson 8: Using Sum/Difference of Two Cubes & Grouping to Find Polynomial Factors

Standard(s)

◆ **2A.7E** Determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping.

Important Vocabulary

- linear factor
- quadratic factor
- greatest common factor
- **sum of cubes**
- **difference of cubes**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson students will factor polynomials using product-sum (ac) method, grouping, and/or sum or difference of cubes.

Opportunities to CFU

✓ INM #1-10
✓ Debrief
✓ Student Practice #5-10

Other Notes to Inform Your Planning

Students will use their understanding of factoring, and number sense to factor using sum/difference of cubes and to factor four term polynomials. This will lead students to use their understanding of factoring to find attributes and graph polynomial functions.

Lesson Look Fors

Look for teachers to…

- ❑ Support students in factoring as many may not remember how to from Algebra 1.
- ❑ Support students with sum and difference of cubes formulas. Students don't need to memorize them.

Look for students to…

- ❑ Factor by grouping, GCF, and the traditional factoring of trinomials.
- ❑ Factor using sum and difference of cubes problems.

Student Know/Do Chart

Students can

Do Factor a variety of polynomials by selecting the best method for the problem at hand.

Do Factor using the sum and difference of cubes.

Know Sum of Cubes,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Know Difference of Cubes,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Date: _____

Lesson 9: Fundamental Theorem of Algebra

Standard(s)

2A.7D Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods.

Important Vocabulary

- **Fundamental Theorem of Algebra**
- **complex conjugate root theorem**
- **conjugate pairs**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator
- Desmos for visuals

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (5 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will verify the Fundamental Theorem of Algebra by finding polynomial zeros and determine the number of zeros based on the degree.

Opportunities to CFU

- ✓ INM #1-5
- ✓ Debrief
- ✓ Student Practice #1-5

Other Notes to Inform Your Planning

This is the only lesson written on the FTA in the updated curriculum. Students will take their understanding of factoring, roots, and finding zeros to apply the Fundamental Theorem of Algebra.

Focus on Disciplinary Literacy

INM & Student Practice

Lesson Look Fors

Look for teachers to…

- ❑ Focus students on the nature of zeros of functions based on the graphs of a variety of polynomial functions.
- ❑ Discuss radical and imaginary zeros along with zeros that students are used to seeing from solving by factoring.

Look for students to…

- ❑ Connect the degree of a polynomial to the number of zeros of the function.
- ❑ Find zeros of a polynomial function to verify the FTA.

Student Know/Do Chart

Students can

Do Find all of the zeros of a polynomial function.

Know The Fundamental Theorem of Algebra states that every polynomial equation of degree n with complex number coefficients has n roots, or solutions, in the complex numbers.

Date: _____

Lesson 10: Graphing Polynomial Functions

Standard(s)

2A.2A Graph polynomial functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$,~~ and ~~$f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.

Important Vocabulary

- extrema
- **turning points**
- zeros
- end behavior
- **multiplicity**
- **even**
- **odd**
- **repeated zero**
- **double root**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (5 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will sketch graphs of polynomial functions by finding zeros (and their multiplicities), y-intercept, symmetry, and end behavior.

Opportunities to CFU

- ✓ After the INM
- ✓ Debrief
- ✓ Student Practice #1-5

Other Notes to Inform Your Planning

Students will use their understanding of attributes, graphs, factoring, finding zeros, axis of symmetry, and more to create graphs of polynomial functions.

Focus on Disciplinary Literacy

INM & Student Practice

Lesson Look Fors

Look for teachers to…

- ❑ Support students in identifying the key features of polynomial functions.
- ❑ Show students the different ways zeros can be represented on the graph of a curve.

Look for students to…

- ❑ Sketch polynomial curves by using the zeros, end behavior, and the nature of even and odd degree polynomial functions.

Student Know/Do Chart

Students can

Do: Use the zeros, their multiplicity, and end behavior to sketch a polynomial function.

Know: The curve of a polynomial function can cross or touch the x-axis at a zero depending on the multiplicity of that zero.

Date: _____

Lesson 11: Solving Polynomials

Standard(s)

- **2A.7E** Determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping.
- **2A.7D** Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods.

Important Vocabulary

- polynomial function
- root
- zero
- factor (noun)
- rational roots
- irrational roots
- **remainder theorem**
- **factor theorem**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator
- Desmos

Lesson Structure:

- Do Now (5 min)
- INM (15 min)
- Debrief (10 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will use a table or graph to find initial zeros, then find remaining zeros when given an equation of a polynomial.

Opportunities to CFU

- ✓ INM #1-2
- ✓ Debrief
- ✓ Student Practice #1-5

Other Notes to Inform Your Planning

The purpose of the lesson is to have students combine all their previous learning to solve polynomials. In earlier lessons, they learned that factors give us the zeros of a polynomial and if you know a factor of a polynomial you can divide it out. If students are given a factor or can find one from a graph, they should first divide it out. After dividing they will see that they are left with a lower degree polynomial that they may be able to solve another way, like factoring or the quadratic formula.

Focus on Disciplinary Literacy

INM

Try not to jump in too soon with telling students how to solve any particular problem. If students struggle, recommend they graph the polynomial look for the x-intercepts. They can also confirm the number and types of zeros by looking at the graph.

Lesson Look Fors

Look for teachers to…

- ❑ Remind students that not all polynomial functions will have zeros that are rational or real.
- ❑ Guide students to draw upon all the skills they have built throughout the unit.

Look for students to…

- ❑ Use long division or the tabular method to divide polynomials by the given factor or by a factor they found from the zeros of the graph.
- ❑ Factor or use the quadratic formula to find remaining zeros of the depressed polynomials they get from dividing out a known factor.

Student Know/Do Chart

Students can

Do: Use a table or graph to find initial zeros, then find remaining zeros of a polynomial function.

Know: The number of complex zeros of a polynomial function is equal to the degree. This means not all of the zeros are always real or rational that can easily be found from a table or graph.

Recommended Unit 7 Success Days Material and Resources

Date: _____

To review topics taught up to 7.5, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review introduction to polynomial functions:

Desmos Interactive Applets:

- Polygraph Activity: Classifying Polynomials

Sample Activities & Tasks:

- TI-Calculator Activity: Applications of Polynomials
- The Largest Box Problem

Content Video Lessons:

- Volume of a Folded Box Problem (Without Calculus)
- Modeling with Polynomials
- Introduction to Polynomials

To review attributes of polynomial functions:

GeoGebra Interactive Applets:

- End Behavior of Polynomial Functions
- Even and Odd Function Tester
- Polynomial Multiplication Visualization
- Polynomial Long Division

Desmos Interactive Applets:

- Adding and Subtracting Polynomials
- Area: Multiplying Polynomials

Sample Activities & Tasks:

- Leading Coefficient Exploration for End Behavior (answers)
- Even and Odd Polynomials Practice (Algebraic/Graphically)

Content Video Lessons:

- Introduction to End Behavior of Polynomials
- End Behavior of Functions and Their Graphs
- Finding Extrema of Polynomials
- Even and Odd Functions
- Even and Odd Polynomials

To review topics taught up to 7.10, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review sum and difference of cubes and other factoring:

Sample Activities & Tasks:

- TI-Calculator Activity: Sums and Differences of Cubes
- Factoring by Grouping Skill Practice
- Sum and Difference of Cubes Skill Practice

Content Video Lessons:

- Introduction to Grouping
- Factoring Polynomials by Grouping
- Factoring Trinomials Using Product-Sum Method
- Factoring Quadratics with Common Factor and Grouping
- Factoring Quadratics with Negative Common Factor and Grouping

To review the Fundamental Theorem of Algebra:

Sample Activities & Tasks:

- Fundamental Theorem of Algebra Skills Practice (answers)
- TI-Calculator Activity: Exploring Polynomials: Factors, Roots, and Zeros
- Graphing Polynomials in Factored Form Practice

Content Video Lessons:

- The Fundamental Theorem of Algebra
- The Fundamental Theorem of Algebra 2
- The Conjugate Pair Theorem
- Graphing Polynomial Functions
- Finding Zeros and Their Multiplicities for a Polynomial
- End Behavior of Graphs of Polynomial Functions

Date: _____

Unit 7 Exam

Standard(s)	Notes for Intellectual Preparation & Lesson Planning
◆ **2A.2A** Graph polynomial functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e~~, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. **Problem #1, 2, 9, 10b** ◆ **2A.7E** Determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping. **Problem #3, 8, 10d** ◆ **2A.7B** Add, subtract, and multiply polynomials. **Problem #5, 6** ◆ **2A.7C** Determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two. **Problem #4, 7, 10c** ◆ **2A.7D** Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods. **Problem #10a**	**Necessary Materials and Pre-Lesson Prep** ▪ Algebra 2 Unit 7 Exam ▪ Assessment Companion for Algebra 2 Unit 7 Exam found on Curriculum Corner **Notes to Inform Your Planning** Review the Unit 7 Exam on Curriculum Corner. Internalize & create an exemplar for the assessment prior to teaching the unit as part of unpacking the unit. Use the exemplar to spar with the answer key provided on Curriculum Corner. Administer the Unit 7 Exam following guidance on the Scope & Sequence to ensure the scoring deadline is met.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.	Including, but not limited to: • Domain and range of the function o Domain – set of input values for the independent variable over which the function is defined o Range – set of output values for the dependent variable over which the function is defined o Representation for domain and range o Domain and range of the function versus domain and range of the contextual situation • Key attributes of functions o Intercepts/Zeros o Symmetries o Maximum and minimum (extrema) • Use key attributes to recognize and sketch graphs • Application of key attributes to real-world problem situations	• Grade Level(s): o The notation $\Re$ represents the set of real numbers, and the notation Z represents the set of integers. o Algebra I studied parent functions $f(x) = x$, $f(x) = x^2$, and $f(x) = b^x$ and their key attributes. o Precalculus will study polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions. o Various mathematical process standards will be applied to this student expectation as appropriate.
2A.7E Determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping.	Including, but not limited to: • Determination of linear and quadratic factors by factorization o Greatest common factor o Difference of squares: $a^2 - b^2 = (a + b)(a - b)$ o Trinomials o Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ o Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ o Grouping methods o Verify factorization by re-multiplying the factors. • Factor using non-algebraic techniques to determine rational roots o Tables o Graphs	• Grade Level(s): o Algebra I introduced factorization of polynomials of degree one and degree two. o Algebra II introduces factorization of polynomials of degree three and degree four. o Various mathematical process standards will be applied to this student expectation as appropriate.
2A.7B Add, subtract, and multiply polynomials.	Including, but not limited to: • Polynomial expression – monomial or sum of monomials not including variables in the denominator or under a radical o Monomial – one term expression, $\frac{2}{3}x^3y^5$ o Binomial – two term expression, $5x^3y - 3x^2z^3$ o Trinomial – three term expression, $2x^2 + 3x + 1$ • Operations with polynomials: addition, subtraction, multiplication o Application of properties of algebra to perform operations o Application of laws (properties) of exponents	• Grade Level(s): o Algebra I performed operations on polynomials of degree one and two. o Algebra II extends operations on polynomials to polynomials of degree three and four. o Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
2A.7C Determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two.	Including, but not limited to: • Division by factoring o Cancellation of common factors in numerator and denominator • Long division o Degree three and four polynomials by degree one and degree two polynomials • Synthetic division o Degree three and four polynomials by degree one polynomials in the form $x - c$	• Grade Level(s): o Algebra I determined the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend. o Algebra II introduces division of degree three and four polynomials by degree one and degree two polynomials. o Various mathematical process standards will be applied to this student expectation as appropriate.
2A.7D Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods.	Including, but not limited to: • Connections between roots and factors o If $x = c$ is a root of a polynomial, then $(x - c)$ is a factor of the polynomial. • Determination of linear and quadratic factors from tables o Identification of roots from a table, x values where y values equal zero o Writing roots as factors • Determination of linear and quadratic factors from graphs o Identification of roots from a graph, x-intercepts or zeros o Writing roots as factors • Determination of linear and quadratic factors by depressing polynomials o Rational root theorem to determine possible roots o Analysis of possible rational roots by synthetic division o Remainder Theorem	• Grade Level(s): o Algebra I introduced factorization of polynomials of degree one and degree two. o Algebra II introduces synthetic division of degree three and four polynomials by degree one polynomials. o Algebra II introduces depression of polynomials to determine roots and factors of the polynomial. o Various mathematical process standards will be applied to this student expectation as appropriate.

VERTICAL STANDARDS

This section details the **progression** of key student expectations (SEs) in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Algebra 1	Algebra 2	Pre-Calculus
• **A.10A** Add and subtract polynomials of degree one and degree two. • **A.10B** Multiply polynomials of degree one and degree two. • **A.10C** Determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and a polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend. • **A.10D** Rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property. • **A.10E** Factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two. • **A.10F** Decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.	• **2A.2A** Graph polynomial functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. • **2A.7B** Add, subtract, and multiply polynomials. • **2A.7C** Determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two. • **2A.7D** Determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods. • **2A.7E** Determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping. • **2A.8A** Analyze data to select the appropriate model from among linear, quadratic, and ~~exponential~~ cubic models. • **NOTE:** Students will also work with higher degree polynomials and their graphs in accordance with vertical alignment and AP readiness goals.	• **(2.G)** Graph functions, including ~~exponential, logarithmic, sine, cosine, rational,~~ polynomial, ~~and power~~ functions and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. • **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational,~~ polynomial, ~~power, trigonometric, inverse trigonometric, and piecewise defined~~ functions, ~~including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing. • **(2.J)** Analyze and describe end behavior of functions, including ~~exponential, logarithmic, rational~~, polynomial~~, and power~~ functions, using infinity notation to communicate this characteristic in mathematical and real-world problems. • **(2.N)** Analyze situations modeled by functions, including ~~exponential, logarithmic, rational,~~ polynomial, ~~and power~~ functions, to solve real-world problems. • **(5.J)** Solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems. • **(5.K)** Solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems.